INTERACTION OF RIGID FOUNDATIONS WITH A BASE THAT DEFORMS NONLINEARLY

INTERACTION ENTRE SEMELLES RIGIDES ET SOL DE FONDATION NON LINEAIREMENT DEFORMABLE

О СОВМЕСТНОЙ РАБОТЕ ЖЕСТКИХ ФУНДАМЕНТОВ И НЕЛИНЕЙНО ДЕФОРМИРУЕМОГО ОСНОВАНИЯ

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SUMMARY. Initial stress-strain relationships, essential for solving the problem on hand, are analyzed. Proposals are set forth for taking into account the rheological soil properties in this solution, on the basis of which a settlement - load curve can be plotted as a function of time. Results obtained in a solution of a problem under plane strain conditions are given in detail. Special consideration is given to the influence of plate width and its embeddment into the base. An evaluation is given of assumptions on which present-day solutions, corresponding to a linearly deforming medium, are based. A relationship is proposed between plate settlement and its load. Here, plate settlement depends nonlinearly on plate width, embeddment and the load on the plate. The disagreement between test data and results of a solution based on the hypothesis of a linearly deforming medium can be explained to a large extent by the nonlinearity of shear strain in soils.

Investigations and Calculation Methods

The theory of a medium that deforms linearly is usually widely used for calculating the stressed state and the settlement of the bases of structures. However, the range of application of this theory is very limited. For this reason, a further development of the deformational theory of plasticity of soils (Botkin,1940) and its application in the design of bases of structures is essential.

The relationship between the components of the stress tensors σ_{ij} and the components of the strain tensors ε_{ij}, according to the deformational theory, is as follows

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \delta_{ij}\lambda\varepsilon_{kk} \quad (1)$$

where μ and λ are scalar functions of the invariants of the stress and strain deviators, whose form was determined experimentally (Shirokov, et al, 1971).

It has been shown that on the basis of physical prerequisites, an equation of the soil state can be written which takes into account the time factor (Zaretsky and Vyalov, 1971). An approximation of the law of distortion upon slowly changing loads in time (aging theory) can be presented in the following form (Zaretsky, 1972):

$$\varepsilon_i = \frac{B\,\sigma_i t}{T + (\sigma_{i(\infty)} - \sigma_i)t} \quad (2)$$

where B and T = parameters; t = time; $\sigma_{i(\infty)}$ = ultimate long-term strength, determined by the relationship

$$\sigma_{i(\infty)} = \Phi(\sigma_i, \sigma, \lambda_\sigma) \quad (3)$$

where λ_σ is the Lode stress parameter.

Equation (2) at $t \to \infty$ passes over into the relationship:

$$\varepsilon_i = \frac{B\,\sigma_i}{\sigma_{i(\infty)} - \sigma_i} \quad \text{or} \quad \sigma_i = \frac{\sigma_{i(\infty)}\,\varepsilon_i}{B + \varepsilon_i} \quad (4)$$

According to Mises-Schleicher

$$\sigma_{i(\infty)} = \Phi(\sigma_i, \sigma) = C_B + \sigma \tan \varphi_B \quad (5)$$

and equation (4) coincides with a proposal of A.I.Botkin (1940). The boundary surface can be presented in another form, that corresponds better with test data (Shirokov, et al, 1971):

$$\sigma_{i(\infty)} = \Phi(\sigma_i\ \sigma, \lambda_\sigma) = (C_B + \sigma \tan \varphi_B)$$

$$\left\{ I + m \left[1 + \frac{(9-\lambda_\sigma^2)\lambda_\sigma}{(3+\lambda_\sigma^2)^{3/2}} \right]^n \right\}^{-1} \quad (6)$$

where m and n are parameters.
In addition to the equation characterizing distortion of the soil upon shear, the law of volume strain of the soil must also be employed. The magnitude of volume strain $\varepsilon_{kk} = 3\varepsilon$ is influenced by the hydrostatic pressure σ, the intensity of the shearing stresses σ_i, and the type of stressed state λ_σ (Zaretsky, 1967). In this paper, soil dilatancy is disregarded, assuming that

$$\text{(a)} \quad \varepsilon = K^{-1}\sigma \quad \text{(b)} \quad \varepsilon = K_0^{-1}\sigma^{1-\nu} \qquad (7)$$

In the first case K is a constant, in the second, K_0 and ν are constants.
The above-stated laws of deformation are employed for solving a problem of the action of a circular rigid plate (Shirokov, et al, 1971) and a strip rigid plate on a soil base (Fig.1).

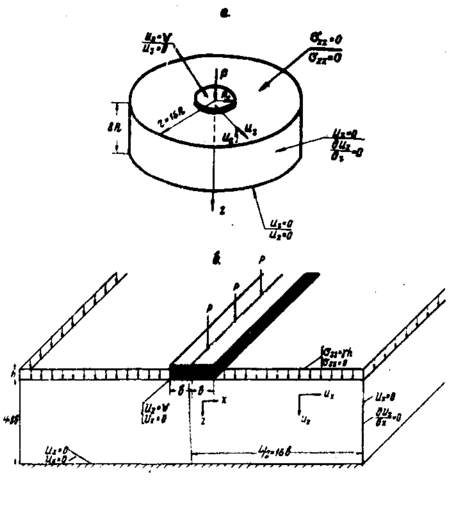

Fig.1.
Calculation Scheme showing the position of the coordinates and the boundary conditions; (a)circular rigid plate,(b)strip rigid plate.

The aim of this investigation is to establish to what degree the results of theoretical solutions approximate test results, when nonlinearity of deformation laws is taken into account, and to obtain an idea of the influence of factors σ_i, σ and λ_σ that determine shear strain in soils.
Derived for the problem on hand is a system of two nonlinear differential equations of the second order with respect to the displacement components (U_z, U_r for the circular plate and U_z and U_x for the strip plate). The boundary conditions at the contact of the rigid (circular and strip) plates with the soil base, correspond to their complete adhesion. Beyond the plates, normal and shearing stresses are absent at the boundary (Fig.1).
For solving this system of equations, a method of variable parameters of elasticity was employed. This method is a modification of the method of elastic solutions (Ilushin, 1948) and consists in solving a system of nonlinear differential equations by means of successive approximations. Moreover, the values of the variable parameters of elasticity μ and λ are known for each approximation. These parameters are computed according to the displacement values and their derivatives with respect to r or x and z from the preceding approximation. The solution of the system of equations at μ = Const and λ = Const is taken as the first approximation. The method of finite differences was used for solving the equations. This method reduces the differential equations for each approximation to a system of linear algebraic equations. Solutions of well-known problems in the linear theory of elasticity, obtained by means of the developed program, indicated the high accuracy of the results.
In working out the algorithm for solving the problem, the dead weight of the soil was included in the equilibrium equation. The solution of the nonlinear problem of the action of the dead weight of the soil was taken as the initial approximation. In this case the time for computation is greatly reduced and the accuracy is improved.

__An Analysis of Solution Results.__

The solution of the problem of sinking a testing plate into a sand base and into a base of cohesive clayey soil having a specific cohesion of 1 kg/cm^2, under conditions of axisymmetric and plane strain, was obtained for the following shear strain relationships:

$$\text{(a)}\ \sigma_i = \frac{C_B\ \varepsilon_i}{0.0075+\varepsilon_i} \qquad \text{(b)}\ \sigma_i = \frac{(0.45\sigma + C_B)\varepsilon_i}{0.0075+\varepsilon_i}$$

$$\text{(c)}\ \sigma_i = \frac{(0.96\sigma + C_B)\varepsilon_i}{0.0075+\varepsilon_i} \qquad (8)$$

$$\text{(d)}\ \sigma_i = \frac{0.96\,\sigma\,\varepsilon_i}{0.0075+\varepsilon_i} \qquad \text{(e)}\ \sigma_i = \frac{0.96\sigma\varepsilon_i f(\lambda_\sigma)}{0.0075+\varepsilon_i}$$

Here f (λ_σ) corresponds to the curly brackets in equation (6). Constants of the laws of deformation, were taken as tan φ_B = =0.96 and B = 0.0075 according to test data for sandy soil with a void ratio e = 0.6 and unit weight of 1.66 g/cm^3.
The settlement calculation for an unsunken strip 2b = 15m indicated that, in contrast to axisymmetric deformation of the soil base

(Shirokov, et al, 1971), upon plane strain the influence of the type of stressed state is greater. The settlement, when using equation (8e), is less by an amount up to 10 per cent, than when equation (8d) is employed at $q \leqslant 0.5$ kg/cm^2 and by an amount up to 24 per cent at $1 \leqslant q \leqslant$ $\leqslant 3$ kg/cm^2. The results of a numerical solution are given below for a problem, employing the equation (8e) for shear strain at $f(\lambda_{\sigma})$ with respect to equation (6). The volumetric compression was taken according to equation (7b), where $K_o =$ $= 60(\text{kg/cm}^2)^{1-\nu}$ and $\nu = 0.32$.

A Minsk-22 computer was used in solving this problem. For each case the base was approximated by a net area, whose dimensions are given in Fig.1 b. The calculations were carried out in the axisymmetric problem for plates arranged on the soil surface only, and in the plane problem for plates, arranged both on the surface and embedded in the base to depths of 3 m and 1.5 m. In this case, the embedding was accounted for as a surcharge $\sigma_{zz} = \gamma h$ at $/x/ \geqslant b$, where h = height of the layer of soil above the level of the foot of the plate. The plate dimensions were taken 1, 2, 3 and 7.5 m, both for the embedded plates, and for those arranged on the soil surface. The calculation results give an idea of the influence of plate size on the character of the stressed-strained state of the soil base.

Equations, obtained as results of numerical calculations of the relationship between the mean pressure q on the foot of the plate and the plate settlement w are given for the axisymmetric problem in Fig.2, and are tabulated for the plane problem. A detailed investigation of a stressed state in an axisymmetric problem has been given in an article (V.N.Shirokov, et al, 1971), and the problem of base deformation, taking into account the time factor, has been described (Yu.K.Zaretsky, 1972). The results of the solution for the plane problem are considered below.

As is evident from the table, an increase in embeddment leads to a decrease in plate settlement. Hence, if these results are treated from the viewpoint of the linear theory of elasticity, it follows that the embeddment of the plate leads to an increase in the modulus of deformation of the base. In accordance with the theory of elasticity, the settlement of a strip plate is proportional to its width. Results of calculations indicate that such a proportion does not exist in a nonlinearly deforming medium. The obtained results enable the disagreement between settlements, predicted on the basis of the theory of elasticity and the actual settlements of large-area foundations, to be explained. Employing a soil model in the form of a nonlinearly deforming body and taking into account the dead weight of the soil, it became possible to reveal this phenomenon.

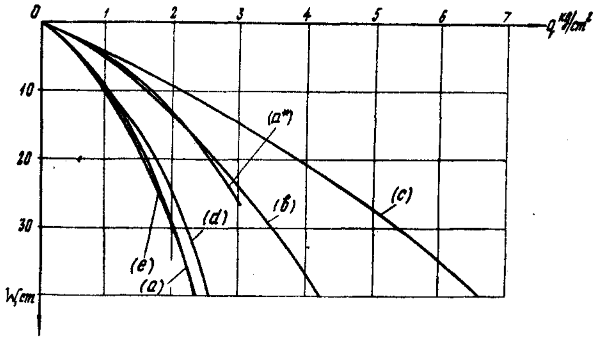

Fig.2.
Settlement w vs load q curves for the assumed deformation laws according to equations (8a), (8b), (8c), (8d) and (8e). Curve (a*) corresponds to equation (8a) at the cohesion value $c_B = 2$ kg/cm^2.

As shown by an analysis, made by Yu.K.Zaretsky, results of numerical calculations revealed good linearization in coordinates q/w - q for plates whose width 2b equals 1, 2, 3 and 7.5 m.

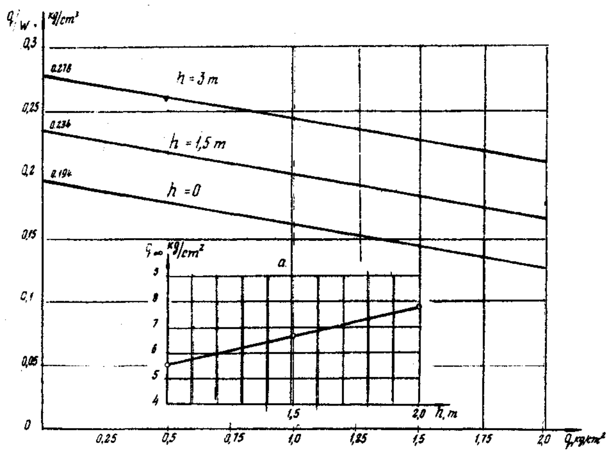

Fig.3.
Strip plate settlement (width 2b = 3.0 m) vs load curves in q/w - q coordinates; (a) ultimate load q_{∞} vs embeddment depth h curve.

As seen in Fig.3, where such linearization is shown for a plate 3 m in width, the calculation results for various embeddment depths can be approximated by parallel straight lines. This means that equation

$$W_{\infty} = A \frac{q}{q_{\infty} - q} \qquad (9)$$

is valid for describing the settlement of a strip plate. Here parameter A is independent of the embeddment depth of the plate (parallelism of lines in Fig.3). From a processing of the results it also follows that the ultimate load depends linearly on the embeddment depth of the plate (Fig. 3a) and on its width. Further, in the general case,

$$q_{\infty} = N_1 \gamma b + N_2 \gamma h + N_3 C, \text{ where}$$

$N_1 N_2, N_3$ depend only on the angle φ_B and the parameter λ_σ, i.e. on the angle of internal friction. It follows from results of the numerical solution that $A = A_0 b = 0.094$ b. Here, the dimensionless coefficient A_0 can be presented as $B/[2(1 - \tan \varphi_B)]$, where B is a parameter of the deformation law /equation (4)/ and φ_B is always less than 1.

Finally, the dependence of the strip plate settlement on the load, its width, the embeddment depth and the deformability characteristics of the soil base can be written as:

$$W = \frac{B}{2(1 - \tan \varphi_B)} \cdot \frac{q\, b}{N_1 \gamma b + N_2 \gamma h + N_3 C_B - q} \qquad (10)$$

Besides, coefficients N_1, N_2 and N_3 also depend on the shape of the plate in the plan view. It follows from equation (10) that the modulus of total deformation of the base is proportional to $(N_1 \gamma b + N_2 \gamma h + N_3 C_B - q)$ to an accuracy up to a constant factor. This means that the modulus of deformation of the base from the viewpoint of the theory of elasticity, increases with the width of the plate and its embeddment, and decreases with an increase in the applied load.

A complete analysis of the stressed-strained state of a base subject to a strip plate was also made. The character of displacement damping along the depth, the change in stresses σ_{zz} along the depth, the isolines of the shear moduli μ are similar to results obtained in solving the problem for a circular plate (Shirokov, et al,1971).

Test results, showing the dependence of the depth of the strata subject to compression on the load and the width of the plate (Fig.4) are of special interest.

Here, it was assumed that the strata subject to compression is limited by a depth, obtained for a half-plane, at which displacements comprise 95 per cent of the settlement. At an increase in load, the compressed strata increases; however, its height does not exceed three widths of the plate. Moreover, the smaller the load, the greater the influence of the plate width.

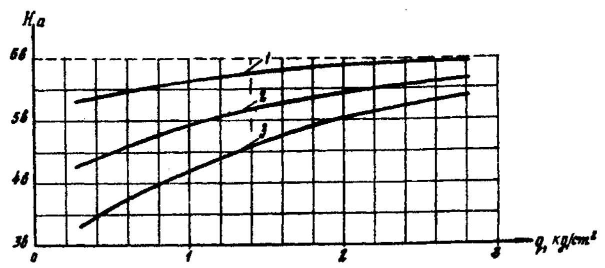

Fig.4.
Depth of compressed zone H_a vs load curves: at a plate width 2b = 1.2 m(1); 2b = 7.5 m (2) and 2b = 15 m (3).

Reaction pressure curves are given in Fig.5. An increase in load, both for embedded and surface plates, leads to a reduction of the "saddle" in the curve, but for the surface plates this tendency is more pronounced.

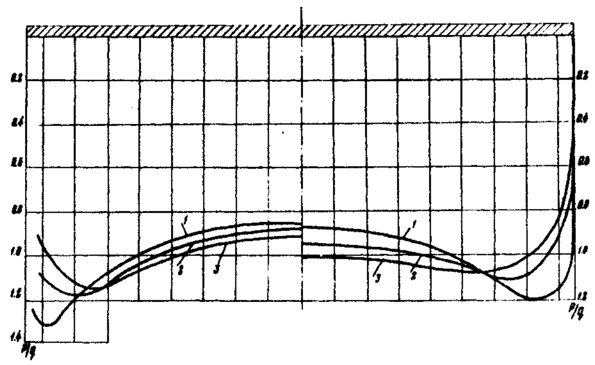

Fig.5.
Reaction pressure curves, whose ordinates are referred to the mean load on the plate. Strip width = 7.5 m; at left for an embeddment h = 3.0 m; at right, - on the surface.
Curve (1): q = 0.5 kg/cm^2; curve (2): q = 1.5 kg/cm^2 and curve (3): q = 2.5 kg/cm^2.

<u>Conclusions.</u> The following conclusions can be reached from an analysis of calculation results for strip and circular testing plates:

1. The relationship between the settlement and the load is nonlinear. An increase of internal friction, at the same values of cohesion and parameter B in equations (8), greatly decreases the settlement for the same load.

2. Upon axisymmetric strain, the influence

of the Lode parameter λ_σ affects the settlement only slightly. However, in plane strain, its influence becomes quite substantial. The more λ_σ differs from -1, the lesser the settlement, all other conditions being equal.

3. The plotting of shear moduli isolines enabled the formation of areas to be revealed near the plate edges with a lower shear rigidity.

4. Calculations showed that, under a plate situated on a sand base, an area is formed whose shear rigidity continuously grows with an increase in load. The boundary of this area does not depend on the load. The obtained changes in deformation properties of the base, in the process of loading, correspond to the prerequisites assumed for the elasto-platic problem, and are experimentally substantiated (Skormin, Malyshev, 1970).

5. Displacements along the plate axis in a nonlinearly deforming soil are qualitatively of the same character, as in a linearly deforming medium. However, damping of the vertical displacements with depth along the plate axis is more rapid. Thus, in the case of a circular plate, making use of the strain relationship (8 e) and having a load q = 1.24 kg/cm^2, compression of a soil layer having a thickness 3R yields 94 per cent of the settlement, and only 6 per cent of the settlement is due to the compression of the underlying soil (calculations were carried out for R = 7.5 m). For a linearly deforming medium, these values are 70 and 30 per cent, respectively.

6. The calculations showed that, with an increase in plate width and its embeddment in a nonlinearly deforming base, the settlement decreases, all other conditions being equal.

7. The discrepancy between test results and solutions, based on the hypothesis of a linearly deforming medium, can be explained, to a considerable degree by the nonlinearity of shear strain in soils, and can be eliminated by generalizing solutions similar to those, obtained in this work.

Strip Plate Settlement W, cm

Width of Plate 2b, m	Load q, kg/cm^2 0.5	1.0	1.5	2.0	2.5	3.0
h = 0						
1.2	1.5	3.2	5.8			
3.0	2.9	6.3	10.2			
7.5	4.8	10.4	16.6	23.8		
h=1.5m						
1.2	1.1	2.4	4.0	5.8		
3.0	2.3	4.9	8.0	11.6	15.5	
7.5	4.0	8.6	13.8	19.7	26.6	34.0
h=3.0m						
1.2	0.9	1.9	3.2	4.7		
3.0	1.9	4.0	6.5	9.8	13.3	
7.5	3.6	7.4	12.0	17.1	23.1	31.2

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