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Study of stress-strain state of two-layer base

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Soil bases are, as a rule, composed of layers with different mechanical properties and the abvisability of considering the non-linear relation between stresses and strains in soils is now universally recognized. The consideration of soil properties, according to the theory of plastic deformation, makes it possible to bring closer together the estimated and experimental results. As may be seen from the experimental data and the results of the solation of some problems the angles of rotation of originally horizontal areas for the zones with considerable deformations of the base below the edges of a rigid loading plate are as great as 45^o. This also generates a need for considering the angles of rotation in equilibrium equations of an element of a volume when defining the variation in size of the linear elements.

Below are given the input equations wherein the non-linearities of two types are taken into account, viz.: physical and geometrical ones what complicates the problem to a considerable extent and impedes the possibility of its analytical solution. The method of finite differences is utilized for the numerical solution of the problem. The two-layer base loaded by the rigid plate is treated under conditions of a plane strain.

The presence of a weak soil layer gives rise to the substantial strains of the two-layer base and the plate settlements what determines the advisability of considering the geometrical non-linearity.

The physical relations of the soil straining are taken as (Krigel H.I.,1973).

a. law of deformation

$$\varepsilon_i = \frac{[B_K + E_K \sigma_{cp}]\,\sigma_i}{C_K + A_K \sigma_{cp} - \sigma_i} \qquad (1)$$

b. law of volumetric straining

$$\varepsilon_o = D_K \cdot \sigma_{cp}^{\alpha_K} + F_K \cdot \sigma_i \qquad (2)$$

where $\sigma_{cp} = \sigma_1 + \sigma_2 + \sigma_3/3$;

$$\sigma_i = \sqrt{\frac{1}{6}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

$$\varepsilon_o = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)/3 \;;$$

$$\varepsilon_i = \sqrt{\frac{2}{3}[(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2}$$

$A_K, B_K, C_K, D_K, E_K, F_K, \alpha_K$ = parameters defined experimentally for each base layer "k", $A_K = tg\,\varphi$ tangent of the angle of internal friction, B_K = coefficient characterizing the non-linear properties of the law of deformation, C_K =parameter of soil cohesion, $D_K \sigma_{cp}^{\alpha_K}$= term taking into consideration the influence of hydrostatic head on volumetric strain (intensity of the volumetric strain growth attenuates with increase of the average stress), E_K considers the influence of the average stress on the deformation (the greater the value $E_K\,{}_{cp}$, the closer the relation to that of linear), $F_K \sigma_i$ =term taking into consideration the influence of tangential stresses on the volumetric strain. The physical relations of the soil straining are taken from the graphs of stabilimetric test $\varepsilon_i = f(\sigma_i, \sigma_m)$

The soil samples were tested for triaxial compression. The cylindrical sample was subjected to the axial, σ_z, and lateral pressures, $\sigma_x = \sigma_y = \sigma_2$. In this case

$$\sigma_m = \frac{\sigma_1 + 2\sigma_3}{3} \;;\; \sigma_i = \frac{1}{\sqrt{3}}(\sigma_1 - \sigma_3)$$

In the general case a set of the input differential equations of mechanics of continuous media taking account of the physical and geometrical non-linearities is written in a tensor form (Novochilov,V.V.,1958). The final strain of an elementary volume of the soil base is completely defined by six components of a strain tensor, ε_{ij}, which are related to the derivatives of projections of a displacement vector, u_i, by Greens relations

$$\varepsilon_{ij} = \frac{1}{2}(u_{ij} + u_{ji} + u_{ni} \times u_{nj}) \quad (3)$$

where i,j,n=1,2,3.

The relative strain, E_{x_i}, of a section, ℓ_{x_i}, parallel to the strain of X_i-axis is determined using the components ε_{ij} by Eq.(4).

$$E_{x_i} = \frac{\Delta \ell_{x_i}}{\ell_{x_i}} = (1 + 2\varepsilon_{ii})^{1/2} \quad (4)$$

Equilibrium equations

$$\left((\delta_{in} + u_{in}) \times \sigma^*_{jn}\right)_j + \frac{V^*}{V} F_i = 0 \quad (5)$$

where i,j,n=1,2,3

δ_{in} = component of Kroneker's unit tensor, V and V^* =volume of the element respectively before and after the strain, F_i =projection of a vector of a specific volumetric force acting at a respective point of a deformed body on the axis of Cartesian system X_i, σ_{jn} =components of the tensor of the generalized stresses related to the actual and relative stresses by

$$\sigma^*_{jn} = \frac{S^*_{xj}}{S_{xj}} \cdot \frac{\sigma^{true}_{jn}}{1 + E_{xn}} \quad (6)$$

$$\sigma^*_{jn} = \frac{\sigma_{jn}}{1 + E_{xn}} \quad (7)$$

where S_{xj} and S^*_{xj} =sizes of the element areas before and after the strain, S^*_{xj}/S_{xj} =coefficient of tche change in size of the areas initially normal to the X_j-axes, E_{xn} =relative linear strain along the X_n-axis, σ^{true}_{jn} true and σ_{jn} - components of the actual and relative stresses along the unit vectors tangential to coordinate lines X_n in the body deformed.

The actual stress is defined as a ratio of the force vector to the area of the element deformed, and the relative one- to the area of the element non-deformed. The relation between the values σ^*_{ij} and ε_{ij} is defined from Henky's ratios

$$\sigma^*_{ij} - \frac{1}{3}\delta_{ij} \times I_{\sigma^*} = 2G\left(\varepsilon^*_{ij} - \frac{1}{3}\delta_{ij} \times I_\varepsilon\right) (8)$$

where $G = II_{\sigma^*} / II_\varepsilon$;

$$I_{\sigma^*} = \sigma^*_1 + \sigma^*_2 + \sigma^*_3 \;;\; I_\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

$$II_{\sigma^*} = \left\{\frac{1}{6}\left[(\sigma^*_1 - \sigma^*_2)^2 + (\sigma^*_2 - \sigma^*_3)^2 + (\sigma^*_3 - \sigma^*_1)^2\right]\right\}^{1/2}$$

$$II_\varepsilon = \left\{\frac{2}{3}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right]\right\}^{1/2}$$

The regularities of the change of the volume, ε_0, and the form, ε_i which are identified with the magnitudes I_ε and II_ε are usually studied in the tests. The values ε_0 and ε_i are expressed in terms of the experimentally defined principal relative elongations E_{x_1}, E_{x_2}, E_{x_3}.

$$\varepsilon_0 = (1 + E_{x_1})(1 + E_{x_2})(1 + E_{x_3}) - 1 \quad (9)$$

$$\varepsilon_i = \left\{\frac{2}{3}\left[(E_{x_1} - E_{x_2})^2 + (E_{x_2} - E_{x_3})^2 + (E_{x_3} - E_{x_1})^2\right]\right\}$$

The values ε_0 and ε_i enter into the regularities of the change of the first and the second invariants of the strain tensor. In case of the plane strain one of the principal elongations, E_i, equals zero. Then from Eqs.(4) and (9) we get

$$I_\varepsilon \approx \varepsilon_0 + \frac{1}{2}\varepsilon_1^2 \;;\; II_\varepsilon \approx \varepsilon_i \quad (10)$$

From Eqs.(10) one may define the change of the strain invariants, I_ε and II_ε as a function of the relative stresses, σ_{ij} using the experimentally defined regularities of the change of ε_0 and ε_i. For a given case the physical equations will define the relations between the components of the tensor of the final strain, ε_{ij} and the relative stresses, σ_{ij}. One should take advantage of Eqs.(7) for the transition to the generalized stresses. In the equations written in a tensor form the summation from 1 to 3 is carried out with the repeating indices. The comma ahead of the index signifies the differentiation of the expression with respect to the variable with the index considered. Eqs.(3), (5) and (8) should be correlated with the given statistical conditions on the base surface. On the boundary between different soil layers one assumes the condition of equality of the movements of soil particles closest to the boundary. The input differential equations and the boundary conditions of the problem are presented in a finite-difference form. The zone of confined dimensions is separated from the base and covered with the irregular network in the nodal points of which the input differential equations are approximated. The bottom and side boundaries of the marked out section of the base are taken at the sufficient distance from the place of the application of the external load to neglect the displacements caused by these loads at the boundaries of the zone in question. The design scheme of the base with the above stated dimensions and the accepted boundary conditions is given in Fig.I.

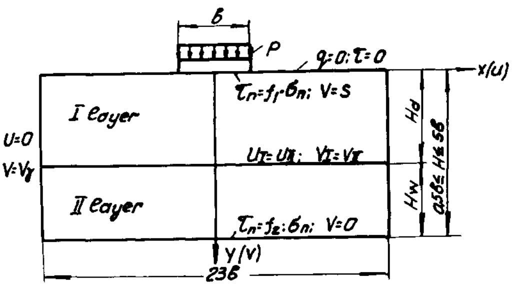

Fig.1. Design scheme of the base

When solving the problem under consideration with respect to the displacements, the set of the finite-difference equations may be represented as

$$[K(U)]\{U\} = \{P\} + \{P^{NL}(U)\} \quad (11)$$

where $[K(U)]$ =rigidity matrix, depending on the nodal movements, built up of the elements expressed in terms of the sum of the volume and shear moduli, $\{P\}$=vector of the summarized external forces where of the components are formed by reducing all the external forces acting on the element to the nodal point load, $\{U\}$ =vector of the summarized displacements, $\{P^{NL}(U)\}$ =vector of the fictitious forces caused geometrically by the non-linear part of the difference equations, e.g. $K(U+U^2)=P$, $K(U)+KU^2=P$ where $K(U)$=matrix element, KU^2= fictitious force.

The solution of a set of the non-linear algebraic equations (11) is effected by the method of successive approximations according to the following recurrent relation:

$$[K(U_{m-1})]\{U_m\} = \{P\} + \{P^{NL}(U_{m-1})\} \quad (12)$$

where m=number of the iteration. Beginning with an original approximation $\{U_0\}$, the iterations are performed up to the convergence on the basis the recurrent relation (12). Checking for convergence is fairly time-consuming since it takes the great number of calculations. Therefore the simplier conditions are adopted as a criterion of the convergence of the iteration procedure (12).

$$mod[(R_B - R_P)/R_B] < r \; ;$$
$$mod[(M_B - M_P)/M_B] < r$$

where mod=sbsolute value, R_B and R_P = total forces of external and reaction loads on the plate, r= given small positive value, M_B and M =moments of external and reaction loads with respect to the central plate axis. Basing on the given algorithm, the program of the numerical solution of the non-linear problem on the computer "EC" has been compiled.

Below is given the study of the geometrical non-linearity effect on the computation results only for the two-layer soil base in the sublimiting state.

A number of researchers, while carrying out the field tests on the stratified bases, made use of the 1x1m plate by 112.6cm in diameter. In this connection, when considering the stress-strain state of the two-layer base with regard to the embedment depth of the weak layer, we made use in calculations of the strip plate width equal to 1 m.

A few types of the two-layer bases were considered with regard to the soil deformability.

The parameters of physical laws (1), (2) for the soil kinds under consideration are adopted as follows:

<u>sand</u>- $A=0,8$, $B=0,0008$, $c=0$, $D=0.0038$ $MPa^{-0.5}$, $E=0$, $F=0.03$ MPa^{-1}, $\alpha=0.5$.

<u>loam</u> $A=0.75$, $B=0.007$, $C=0.04$ MPa, $D-0.0015$ $MPa^{-0.5}$, $E=0$, $F=0.01$ MPa^{-1}, $\alpha=0.67$.

<u>silt</u> $A=1.64$, $B=0.03$, $C=0.01$MPa, $D=0.013$ $MPa^{-0,5}$, $E=0.85 MPa^{-1}$, $F=0.2$ MPa^{-1}, $\alpha=0.5$.

At the Institute of Bases and Underground Structures there were carried out experimental studies under the guidance of Vyalov S.S. (Vyalov,S.S.,1978) on the weak clayey soil with water content of 36%, as another variety of the two-layer base, with the following parameters: $A=0.68$, $B=0.0173$, $C=0$, $D=0$, $E=0$, $F=0$, $\alpha=0$. The upper layer represented the medium-grain sand with density $\gamma=1.61$ g/cm^3 having the following physical parameters: $A=0.9$, $B=0.005$, $C=0$, $D=0.00027$ $MPa^{-0.5}$, $E=0$, $F=0.007$ MPa^{-1}, $\alpha=1.8$.

To compare the results with the test data we took the combinations of thicknesses of the layers making up the two-layer base the upper dense layer (sand) was variable from 0.5 b to 4 b, and the thickness of the underlying weak layer (silt) varied between 0.5 b, b and 2 b. The thickness of the upper layer ranged therewith from 0.5 b to 4 b for each magnitude of the underlying layer thickness.

There were studied the influence of boundary conditions,plate width, depth of embedment of the weak soil layer and its thickness upon the stress-strain state of the two-layer base.

The stratified non-uniformity governs the distribution of stresses in the foundation base different from that in the uniform base. In this case the more is a difference in strain characteristics of the layers, the greater is a discrepancy in distribution of the stresses. The stress-strain state of the two-layer base under the rigid strip plate is dependent on the boundary conditions over the contact surface of the plate and that of the underlying layer, the plate width and the load value,the deformability of layers characterized by a "two-layer" parameter, n, (ratio of moduli of total deformability of soils under similar loading and plate width), the height of the upper dense layer and the thickness of the underlying weak layer. Depending on these factors, the paper considers the two-layer base with regard to the embedment depth of the weak layer, its thickness and deformability.

Below are given the results of calculations of the two-layer base in case of the same loading $P=0.1$ MN/m^2 with the plate width, b=1m, with the different "two-layer"parameter. The thickness of the weak layer was taken to be equal to b and that of the dense sandy layer varied from 0.5b to 4b under the following boundary conditions: there was taken on absolute agglutination at the plate contact with the soil base and a complete slippage at the interface of the weak layer and the rigid half-space.

Fig.2 illustrates the character of the plate settlements, it shows the relation of the plate settlement versus the height of the dense sandy layer. If the thickness of the dense layer is small,the plate settlement is great. However, as the thickness of the dense layer increases up to 2.5b the plate settlement goes sharply down, but the further increase in thickness of the soil dense layer does not lead to the appreciable decrease in settlement of the plate. From this it follos that the optimum thickness of the weak layer is 2.5b for the two-layer base.

Let us consider the character of interaction between the dense sandy and weak silty layers. The analysis of the law of distribution of vertical stresses along the symmetry axis in depth (Fig.3) shows that in case of small thick-

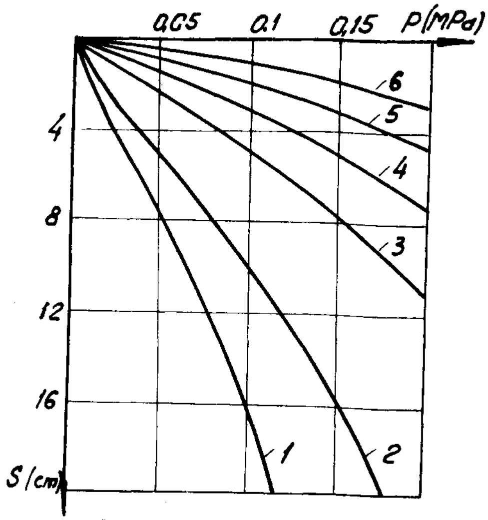

Fig.2. Relations of the plate settlements (S) versus the load (P) for various thicknesses of the upper layer (H_d)
1-thickness of the silty layer if b similar for other cases;2-thickness of the upper layer 0.5b; 3-the same for 2b; 4-the same for 3b; 5-the same for 4b;6-thickness of the sandy layer of 5b.

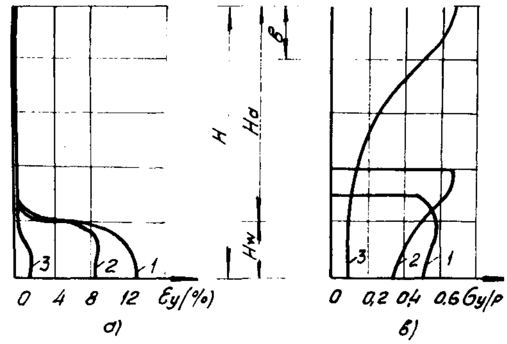

Fig.3. Distribution of relative strains (a) and stresses (b) across the vase depth versus the upper layer thickness H_d with the sim·lar thickness of the weak layer H_w=b and the load P=0.1 MPa
1-thickness of the upper layer of 0.5b;2-the same for b;3-the same for 2b;4-the same for 4b.

nesses of the upper dense layer (less than 0.5b) for all the adopted thicknesses of the weak layer (0.5b, b, 2b) the dispersion of stresses does not come about and the weak layer is subjected to the relatively great invalue stresses. The graphs (Fig.3) show that in case of the thicknesses of the upper dense layer of the order of 0.5b the substantial concentration of the vertical stresses in the weak layer is evident, as a result the considerable strains of the weak layer under the plate take place. Analyzing the regularities of the variation of the shear moduli in the foundation base, one may draw a conclusion that the relatively rigid to shear zone is formed in the dense layer under the plate, and the attenuated to shear zone is observed in the layer under the plate edges. The increase in thickness of the upper dense layer results in reducing the stress concentration in the weak layer (the ratio of the max vertical stresses on the symmetry axis in the weak layer and that in the dense layer is close to unity in case of the identical thicknesses of the upper and underlying layers of 0.5b, this ratio is equal to 0.6 with the thickness of the underlying layer of 0.5b and that of the upper one of 1.5b,and the ratio in question is 0.2 with the thickness of the upper layer of 4b). From these results it follows that the distributive ability of the upper layer becomes substantial when the thicknesses of this layer are more than 2b and the weak layer is subjected to the relatively small stresses resulting in small strains of such layer (Fig.3). The vertical stresses in the weak layer go down with the increase in thickness of the upper dense layer and when the thickness of the upper layer is more than 2 b the vertical stresses transferred to the weak layer change slightly. The vertical strains do not change substantially as well (Fig.3). The character of the variation of the shear moduli in the foundation base shows that there are formed a clearly defined rigid core extending in the base to a depth of 0.5b when the thicknesses of the dense layer are more than 2b. The weak layer is subjected to a practically uniform pressure and the shear moduli are virtually constant. Compared to the one-layer base, the vertical reaction pressures have greater concentration in the two-layer base under the plate edges. The concentration of the vertical reaction pressures under the plate edges is the most marked for the small thicknesses of the upper dense layer.

The concentration of the vertical pressures under the plate edges is relieved with an increase thickness of the upper dense layer, for the thickness of the dense layer more than 2.5b the pressure distribution under the plate becomes practically the same as for the one-layer base. The growth of the weak layer thickness results in on increase of its effect on the stress-strain state of the two-layer base. For example, with the identical thicknesses of the upper and underlying layers of 0.5b the plate settlement are equal to 5.0cm with the thickness of the weak layer of b the settlements are 10.5cm, and with that of 2 b the settlements are 16.8cm. The growth of the settlements is not substantial for the weak layer thickness more than 2 b. The calculation results effected for the weak layer thicknesses of 0.5b and 2b (See Fig.4) show that the influence of the weak layer of the given sizes do not come appreciable for the dense layer thicknesses over 2b and 4 b.

From the analysis of the general graph of the relation of the settlement versus the thickness of the sandy layer with the same soil parameters one can see that the stress-strain state of the two-layer base is dependent on the deformability of the layers making up the base.

On the theory of linear elasticity for defining the modulus of the soil general deformability one makes use of the equation

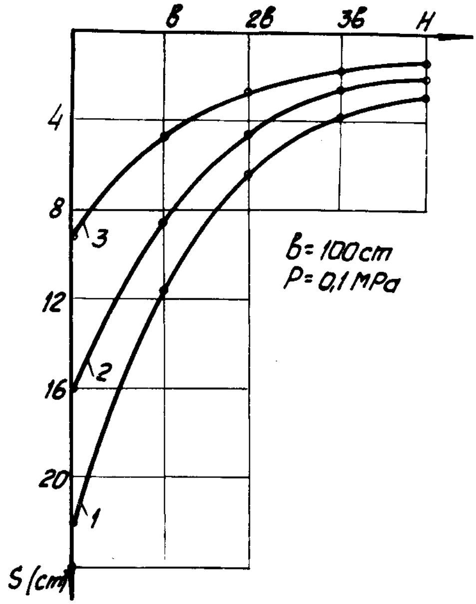

Fig.4. Plate settlements S versus thickness of the upper dense layer H_d with various thicknesses of the underlying layer H_w.

1-thickness of the underlying layer H_w=2b; 2-the same for b; 3-the same for 0.5b.

$$E = \frac{\omega p b (1-\mu^2)}{S} \qquad (13)$$

where S=total plate settlement, ω= coefficient of the shape of the foundation base area, b=width of the base, p=specific soil pressure, μ =Poisson's ratio. In the non-linear solution the physical parameter E can integrally take account of the soil strain properties of the base under the same loads and with the same plate dimensions. For the linearly deformable medium the "two-layer" parameter n is equal to

$$n = \frac{E_1(1-\mu_2^2)}{E_2(1-\mu_1^2)} \qquad (14)$$

where the indices 1 and 2 relate respectively to the upper and underlying layers. From Eqs.(13) and

and (14) with the identical values of w, P and b it follows that $n = \frac{S_2}{S_1}$, i.e. n is equal to the ratio of the settlements. For the non-linearly deformable soils the parameter n can also be calculated as the ratio of settlements for the uniform base, viz.: S_1 for the soil of the upper layer with the thickness nonlimited and on the surface of which the plate is placed to S_2 for the soil of the underlying layer on the interface of which the same plate is located but the is surcharged by the weight of the upper soil layer.

Consideration has been given to 4 kinds of the two-layer bases with different values of the parameter n=2,10,15,25. Fig.5 illustrates

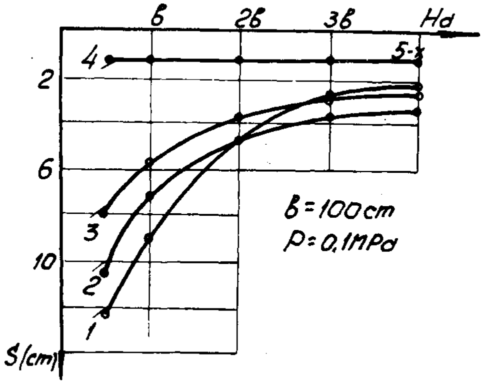

Fig.5. Plate settlements S versus thickness of the upper layer with different parameter n.
1-n=25; 2-n=15; 3-n=10; 4-n=2
5-settlement of the plate on the one-layer sandy base, when H=5b.

the diagrams of the relation between the plate settlements and the height of the dense layer H_d and the value n as an integral index the layer thickness of b.

The analysis of the diagrams of Fig.5 shows that the plate settlements are sharply increased with the growth of n and attenuation of the influence of the weak layer with respect to n is different. When n=2, the plate settlements with the same load on the two-layer base are practically coincident with that on the one-layer sandy base 5b in thickness. While increasing n, the influence of the weak layer grows and the respective thickness of the dense layer, beginning with which the change of the plate settlements becomes insignificant, in the case in point equalts 2,5 b.

CONCLUSIONS

The calculation of the stress-strain state of the two-layer base under the strip plate shows that the influence of the weak layer on the state in question depends on the embedment depth, the thickness, the deformability of the layers making up the base and the load value. The presence of the weak layer, when n 20, results in growth of the plate settlements, however, the lower is the location of the interface, the lesser is the influence of this layer, and the most optimum height of the dense layer, from the standpoint of settlement, is equal to 2.5b. The settlements do not decrease substantially with the further increase in thickness of the upper layer. The settlements drop three times off with the increase in thickness of the upper dense layer from 0.5b to 2,5b, and the plate settlements go 1.3-1.4 times down with the change of the thickness from 2.5b to 4b. In this case the reaction pressures under the plate have the greater concentration under its edges in comparison with the one-layer base. The most intensive influence of the weak layer comes about with its thickness up to 2b, and the change of the settlements and distribution of the reation pressures occur to a lesser degree with the further increase in thickness of the layer. With the small depth of the weak layer embedment one observes the growth of the vertical stresses under the plate edges in comparison with the one-layer base. The influence of the weak layer on the distribution of the reaction pressures becomes insignificant with its depth of embedment of the order of

2.5b. The stratification of the base influences in a greater extent on the stress-strain state, when n 10.

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