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Certain Problems in the Theory of Limit Equilibrium of Loose Media

The history of the development of the theory of limit equilibrium is wellknown and it is not my aim to expound it here. It is necessary only to point out that its development, in the main, is due to works of Soviet scientists, and, primarily to those of V.V.Sokolovsky, who, beginning with 1939, published a number of fundamental papers on this subject, and to V.G.Berezantsev, whose work concerned mainly, the axisymmetric problem. Those who wish to acquaint themselves with the initiation and the development of this field of mechanics should turn to the Works of V.V.Sokolovsky, N.A.Tsytovich, V.G.Berezantsev, S.S.Golyshkevich, and M.I.Gorbunov-Possadov.

The statement of problems in the theory of limit equilibrium is well known: each point of the region under consideration, consisting of loose media, is in a limiting stressed state. The magnitude and the direction of the load are given on one part of the boundary; on another part, only the direction is given, and the magnitude is sought for. There can be other versions: on a part of the contour the magnitude and the direction of the load can be specified, and the configuration of another part of the contour can be sought for. The sought for part can be either entirely without load, or with a load whose magnitude and direction are specified. A load of larger magnitude acting on a part of the contour, is called the dead load. A load of lesser magnitude, acting on the other part of the contour is called the surcharge.

The dual solution of these problems is well known, and is due to the fact that the equation of limit equilibrium is quadratic with respect to the stress components. Thus, if on one part of a contour the magnitude and the direction of the load are specified, and on another part of the contour only the direction is specified, we can obtain two values of its magnitude- one less, and the other more than the specified value. It should be noted that it is not always possible to obtain a solution for a problem of the theory of limit equilibrium. If this is due to the specified combination of boundary conditions (for example, the problem does not have a solution for every inclination of the load, which is to be found), then it usually becomes necessary to resort either to discontinuous solutions, or to obtain a solution under the assumption that a limiting state has not been reached in a part of the region.

An especially important achievement in the theory of limit equilibrium of loose media proved to be a consideration of the stressed state of the so-called special singular point. Credit for this must be paid to V.V.Sokolovsky, who employed the solution of Prandtl's problem for this purpose. The stresses are multivalued at this point. They depend on the angle of approach to the special singular point and vary with the intensity, corresponding to the load within the limits of that part of the contour, where it is specified both in magnitude and direction, to the intensity, which is to be found and is specified only by direction. On the characteristic plane, the special singular point develops into a line.

Since most problems, owing to the nonlinearity of the initial system of equiations, cannot be solved in the closed form, it became necessary to use the method of finite differecnes for integrating this system, which is of the hyperbolic type. The system has two families of real characteristics and integration was carried out with respect to these characteristics thereby simplifying the calculations. Solution of these problems became much simpler in practice when use was made of electronic computers.

The principal problems in the theory of limit equilibrium of a ponderable loose medium (one possessing friction and cohesion) are the following: determination of the bearing capacity of foundation bases, the pressure exerted by the soil in retaining walls, the stability of slopes of a given profile, the outline of the limiting stable slope, the stability of collapse arches, etc.

The application of the method of finite differences for such solutions enables us to deal with lamellar media, anisotropic media, the influence of hydrodynamic forces and other complicating circumstances.

As a rule, all problems dealt with in the statics of loose media are statically determinate, since the contour conditions are specified by stresses. Of practical importance, however, are the cases in which, not only the stresses are given, but some of the deformations as well, that is, cases of combined boundary conditions. Here the statical determinacy is lost, and it becomes necessary to resort to conditions related to the deformations. Viscous flow is considered to be typical of a loose medium in a state of limiting stress. Therefore, it is not the deformations themselves that are determined, but their rates. The deformation rate tensor is usually considered to be coaxial to the stress tensor. The concept of a plasticity potential is introduced. This is a certain function, whose partial derivatives with respect to the stresses, are proportional to the rates of deformation. The plasticity potential is set up analogous to the strength condition. The establishment of such a concept enabled us to deal with the kinematic aspect of the problems being considered, and also facilitated the comparison of the theoretical and experimental results. The introduction of the plasticity potential in the form of a strength condition enabled the direction and magnitude of the deformation rate vectors to be determined for each point of a loose medium in a state of limiting stress as a function of the rate of boundary displacement. Also found was the necessary condition for decompacting (loosening) a loose medium upon shear. This was confirmed experimentally, the amount of loosening being determined in terms of the angle of internal friction. Upon shear, the volume remained unchanged only for media with no friction.

Making use of the plasticity potential hypothesis, R.T.Shield (1953) solved the problem of soil bulging in a foundation base. This solution was thorughly analysed by Yu.I.Solovyev (1969). A special article (Malyshev,1971) was devoted to the question of the interpretation of the lines obtained in an experiment conducted by the photographic fixation method. It was shown in this article that the lines seen on the photographs are not the slip lines, but the envelopes of the total displacement vectors. This article also proposed an expression for the plasticity potential, which takes into account a certain experimental fact: the existence of a so-called critical porosity at which shear occurs without a change in volume though the angle of internal friction is not equal to zero.

Solutions of the theory of limit equilibrium of loose media have been experimentally tested in many cases. Frequently, a discrepancy was found between bearing capacity values determined theoretically and experimentally. Here consideration must be given to important factors associated with the procedure for determining the design strength characteristic of loose soil the angle of internal friction- as well as the lack of correspondence of the boundary conditions for the theoretical solution and the experiments. The first of these factors has become sufficiently clear: it turned out that the angle of internal friction should be determined at the same Lode strain parameter as in the problems. Otherwise, it is necessary to recalculate the angle of internal friction (Malyshev, 1968) according to formulas, increasing its value, since the value of the Lode strain parameter for the case of plane strain is greater than for triaxial compression, that is, greater than -1 (minus one). The influence of the boundary conditions is especially distinctly evident from the experiments of V.K.Fedorov (1971) which established that for a uniformly distributed load on the plate (a flexible load, no elastic core is formed),
two thirds
the bearing capacity was 2/3 of that determined for a rigid

does not occur throughout the whole region when the bearing capacity is exhausted, has been put a long time ago (Gorbunov-Possadov, 1962). This was confirmed by experiments showing the existence of an elastic core under the plate, that is, a zone not in the limiting state (Malyshev, 1953). Therefore, in practicable calculations, it is necessary to obtain a solution of the combined elasto-plastic problem. Such a solution, however, would be unreliable without taking into account the kinematic picture of the phenomenon, that is, without consideration for the displacement within the limits of the zones, and also the development of the zone of the limiting state with an increase in load. Unfortunately, the solution of the problem, when thus stated, is associated with a number of arbitrary factors in the statement itself and with difficulties in calculation even if an electronic computer is available because this problem is one with a moving boundary. One possible way to avoid these difficulties is to use the solution for a nonlinearly deforming continuous medium (Shirokov, et al. 1970) However, here it is necessary to introduce a certain degree of idealization, assuming that a limiting state occurs only for infinitely large shear strains.

In the foregoing we dealt with results obtained mainly for the case of plane strain. The case of axisymmetric deformation (Berezantsev, 1953) has also been investigated in detail. To do this, however, required the concept of complete looseness, that is, the equality of the intermediate and minimum principal stresses because otherwise the statement of the problem would become more difficult. The three-dimensional problem of the theory of limit equilibrium practically the most important of these problems, is unfortunately, not get sufficiently clear in its formulation. Only separate investigations connected with this problem have been conducted, for example by Goldstein (1969).

A number of well known and practically essential results have been obtained in the theory of limit equilibrium of loose media. Some of these have been mentioned in the foregoing. However, in order to perfect methods of calculation and to bring their results closer to those of experimental observations, it would be expedient in the future to consider many more questions that have not yet been answered satisfactorily. These include, for instance, the following.

1. It is necessary to investigate the questions concerning the kinematics of a loose medium in a state of limiting stress more thoroughly, as well as the application of experimentally confirmed plasticity potentials of various kinds. It is necessary to establish the validity of the principle of coaxiality of the stress and deformation rate tensors in the limiting state and whether the principal axes of deformation do not turn in approaching the limiting state.

2. It is necessary to investigate the extent to which the initial stressed state of the soil mass influences the bearing capacity and limiting loads. It is known (Florin,1948) that this initial state may vary in a wide range and depends upon natural historical processes of soil strata formation, while the coefficient of lateral pressure of the soils in their natural occurrence may exceed unity.

3. Especial attention should be paid to the most essential practically, but sufficiently complex, three-dimensional problems of the theory of limit equilibrium. For this purpose, a closed system of equations is to be clearly formulated for determining the state of limit equilibrium in the three-dimensional case and the method of its solution is to be indicated.

4. It is necessary to turn attention to a certain arbitrariness in formulating problems of limit equilibrium, due to the fact that the boundary conditions are not always clear and to the lack of a unique solution. This is especially important for the more complex problems, for instance in those in which we find several singular points (soil pressure on parallel walls, etc.).

5. Combined elastoplastic problems are of especial interest. To obtain their solution it is necessary to deal with regions in limiting stresses states having moving boundaries which are frequently of complex configuration with boundary conditions of a combined type.

6. It is necessary to improve the solution for lamellar and anisotropic media, since in practice homogeneous media are less frequently encountered.

7. Along with the derivation of exact solutions, within the scope of the theory of limit equilibrium of loose media, it is of practical importance to obtain engineering solutions for a number of problems in soil mechanics which would establish the limiting values of the forces that can permissibly be applied to soil masses, as well as the strain for loade approaching the limiting values.

8. It is absolutely necessary to continue with further experimental reasearch, associated both with the establishing of the necessary strength characteristics for loose media, and with the modeling of a number of practical problems.

Those enumerated in the foregoing do not cover all the problems that remain to be solved there are many more. There is no doubt that in the future many more interesting and practically important results will be obtained which are based on the principles of the theory of limit equilibrium of loose media.